Membrane Characterization with Model-Based Design of Experiments

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Abstract

Membrane characterization provides essential information for the scale-up, design, and optimization of new separation systems. We recently proposed the diafiltration apparatus for high-throughput analysis (DATA), which enables a 5-times reduction in the time, energy, and the number of experiments necessary to characterize membrane transport properties. This paper applies formal model-based design of experiments (MBDoE) techniques to further analyse and optimize DATA. For example, the eigenvalues and eigenvectors of the Fisher Information Matrix (FIM) show dynamic diafiltration experiments improve parameter identifiability by 3 orders of magnitude compared to traditional filtration experiments. Moreover, continuous retentate conductivity measurements in DATA improve A-, D-, E-, and ME-optimal MBDoE criteria by between 6 % and 32 %. Using these criteria, we identify pressure and initial concentrations conditions that maximize parameter precision and remove correlations.

Keywords: Membranes, Design of experiments, Parameter Estimation, Dynamic Modelling, Diafiltration

1. Introduction

Membrane processes have shown promise for addressing the critical needs for sustainability and energy efficiency. Recent material design to achieve separations of similar-sized molecules has evolved in the directions of precisely controlling the nanostructure of membranes and identifying chemical functionalities which accentuate desired transport properties (Hoffman and Phillip, 2020; Sadeghi et al., 2018). A detailed understanding of the underlying thermodynamic and transport phenomena can elucidate the molecular interactions and mechanisms that affect the macroscopic transport properties of the membrane (Geise et al., 2014; Yaroshchuk et al., 2018). Motivated by this need, the development of membrane characterization techniques that explore the dependency of membrane performance on feed conditions can greatly accelerate the development of materials (Ghosh et al., 2000). In addition, membrane characterization that elucidates underlying mechanisms provides essential information for scale-up, design, and optimization, facilitating the development of separations.

Design of Experiments (DoE) methods optimize computational and physical experiments to maximize the information gain and to minimize time and resource costs. Classical 'black-box' (a.k.a. factorial, response surface) DoE approaches, which decide the best design by the input-output relationship, does not (directly) incorporate membrane science knowledge; in contrast, model-based DoE (MBDoE) leverages high-fidelity models constructed from underlying physical principles that describe the experimental system (Franceschini and Macchietto, 2008). The information collected from experiments can be applied to discriminate between scientific hypotheses, posed as mathematical models, and to improve the precision of parameter estimation. However, to date, MBDoE has not been applied to membrane characterization techniques.

Guided by data analytics, Ouimet et al. (2021) developed a diafiltration apparatus for high-throughput analysis (DATA) to address the limitations of current membrane characterization methods, e.g., time-consuming experimental campaigns and parameter non-identifiability. In this paper, we use MBDOE and FIM-based analysis to mathematically quantify the improvements reported by Ouimet et al. (2021) and further refine the experimental conditions needed in DATA to characterize membrane transport properties and discriminate between possible transport mechanisms.

2. Mathematical model, materials, and methods

In the dynamic diafiltration experiments described by Ouimet et al. (2021), a concentrated diafiltrate is continuously injected into a stirred cell under applied pressure, permeate is collected in several scintillation vials with the mass of the sample vial, m_v, permeate concentration, c_v, and retentate concentration in the stirred cell, c_f, measured. Using these measurements, three model parameters - hydraulic permeability, L_p , the solute permeability coefficient, B that correspond to the membrane transport properties, and the reflection coefficient, σ, that depends on the thermodynamics of the membrane-solution interface - are estimated via weighted least-square nonlinear regression (Eq. (1) where $\boldsymbol{\theta} = \{L_p, B, \sigma\}$). These parameters are related to the volumetric flux of water, J_w, and the molar flux of the solute, J_s, across the membrane in Eq. (2).

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_i w_{m_{v,i}} \left(m_{v,i} - \hat{m}_{v,i}\right)^2 + \sum_j w_{c_{v,j}} \left(c_{v,j} - \hat{c}_{v,j}\right)^2 + \sum_k w_{c_{f,k}} \left(c_{f,k} - \hat{c}_{f,k}\right)^2 \tag{1}$$

$$J_w = L_p(\Delta P - \sigma \Delta \pi), \;\; J_s = B \Delta c \tag{2}$$

The diafiltration apparatus, the differential-algebraic equations (DAEs) model, the data, and the regressed parameters values, i.e., L_p = 3.90 L·m^{-2}·h^{-1}·bar^{-1}, B = 0.29 μm·s^{-1} and $\sigma = 1$ are described by Ouimet et al. (2021). Three key design decisions, the diafiltrate concentration, c_d, the initial feed concentration, $c_f(0)$, and the applied pressure, ΔP may be optimized to maximize the precision of the estimated parameters from dynamic diafiltration experiments.

3. Fisher Information Matrix (FIM)

The Fisher Information Matrix (FIM), $\mathbf{M}$, measures the information content of measurements and is defined as the inverse of the posterior covariance matrix $\mathbf{V}$, Eq. (4), ignoring the prior information (Franceschini and Macchietto, 2008). Here, $v_{m,rs}$ is the rsth element of the $N_y \times N_y$ inverse matrix of measurements error. $\mathbf{J}_r$ is the sensitivity matrix of output y_r sampled at times t_s and evaluated at nominal parameters values $\hat{\boldsymbol{\theta}}$ and specified experimental design conditions $\boldsymbol{\phi}$.

$$\mathbf{M} = \left[\mathbf{V}(\hat{\boldsymbol{\theta}}, \boldsymbol{\phi})\right]^{-1} = \sum_{r=1}^{N_y}\sum_{s=1}^{N_y} {v_{m,rs}}^{-1}\mathbf{J}_r^T\mathbf{J}_s\,, \qquad \mathbf{J}_r = \begin{bmatrix} \left.\frac{\partial y_r}{\partial \hat{\theta}_1}\right|_{t_1} & \cdots & \left.\frac{\partial y_r}{\partial \hat{\theta}_m}\right|_{t_1} \\ \vdots & \ddots & \vdots \\ \left.\frac{\partial y_r}{\partial \hat{\theta}_1}\right|_{t_n} & \cdots & \left.\frac{\partial y_r}{\partial \hat{\theta}_m}\right|_{t_n} \end{bmatrix} \tag{3}$$

MBDoE techniques increases parameter precision by minimising a metric of $\mathbf{V}$ or equivalently maximizing a metric of $\mathbf{M}$. A-, D-, E-optimal experimental designs correspond to minimising the trace, the determinant, and the maximum eigenvalue of $\mathbf{V}$, respectively (or maximizing the trace, the determinant, and the maximum eigenvalue of $\mathbf{M}$). The determinant and trace of the covariance matrix $\mathbf{V}$ can be interpreted as the volume of the covariance ellipsoid under feasible experimental conditions, while the maximum eigenvalue represents the size of the major axis, minimizing them reduces model parameter uncertainty. Additionally, the modified E-optimal (ME-) criterion minimizing the condition number of $\mathbf{M}$ which is defined as the ratio of the largest to the smallest eigenvalues, removes the correlation of parameters.

4. Results and discussions

4.1. Diafiltration experiment enables identification of all model parameters

Table 1 compares the FIMs and their eigen decompositions for experiments in both filtration (F) and diafiltration (D) modes as reported by Ouimet et al. (2021). The analysis of each mode considers one experiment with continuous data collection from the inline conductivity probe (M1) and one experiment encompassing only the initial and final retentate measurements (M2). The elements of the FIMs are one order of magnitude larger for diafiltration (D) than filtration (F) experiments. This shows diafiltration experiments contains more useful information to infer the model parameters. Moreover, analysing the eigenvalues and eigenvectors indicates which parameter can be precisely estimated through experimental design. For example, the minimum eigenvalue of filtration (F) M1 is 4.93E+05; the corresponding eigenvector is predominantly in the direction of model parameter σ. Under the same mode, the largest eigenvalue, 4.71E+09, corresponds the eigenvector in the direction of L_p. This difference, 4 orders of magnitude, indicates that a filtration experiment alone is unable to precisely estimate σ. In contrast, for diafiltration (D) mode, the eigenvalues whose corresponding eigenvectors in the direction of σ, 8.53E+10 in M1 and 8.18E+10 in M2, become the largest ones. Moreover, the smallest eigenvalues for diafiltration mode are 2.17E+08 (M1) and 1.96E+08 (M2), which are 3 orders of magnitude larger than the smallest eigenvalues for filtration mode. This difference indicates that diafiltration experiments are better suited to precisely estimate all three model parameters. Both findings are consistent with the sensitivity analysis results from Ouimet et al. (2021).

4.2. Additional retentate measurements improve parameter precision

Ouimet et al. (2021) show that measuring the retentate concentration is necessary to identify a converging set of parameters. We now use MBDoE to quantify the information content of the additional measurements. Recall M1 in Table 1 considers inline conductivity probe measurements for the retentate whereas M2 omits these measurements and only considers initial and final retentate measurements. Table 1 shows elements and eigenvalues of FIMs of M1 are always larger than M2, which shows the additional data increases the precision of the estimated parameters for both modes. Furthermore, for diafiltration, Table 2 shows 6 % and 32 % reduction in terms of the volume of the confidence ellipsoid from A-, D-optimal criteria, respectively, 11 % reduction in terms of the uncertainty of the least confident parameter (B) from E-optimal, and 6% improvement in the ME-optimal criterion which measures parameter correlation. Similarly, Table 2 also shows 8 %, 17 %, and 2 % improvements from A-, D-, E-optimal criteria, respectively, in filtration experiments. However, the 6 % worsening of the ME-optimal criterion, indicates collecting additional data in filtration mode increases the correlation of the estimated parameters.

Table 1. FIM, eigenvalues and eigenvectors of FIM are calculated in both filtration (F) mode and diafiltration (D) mode. Model M1 includes inline conductivity probe measurements while models M2 includes only the initial and final retentate measurements.

Mode	Model	FIM (×1e9)			Eigenvalues	Eigenvectors		
		L_p	B	σ		L_p	B	σ
F	M1	4.67	-0.01	-0.40	4.93E+05	-8.57E-02	-8.50E-03	-9.96E-01
		-0.01	0.02	0.00	1.74E+07	1.80E-03	1.00E+00	-8.70E-03
		-0.40	0.00	0.04	4.71E+09	-9.96E-01	2.60E-03	8.57E-02
	M2	4.34	0.01	-0.37	4.85E+05	8.60E-02	3.30E-03	9.96E-01
		0.01	0.02	0.00	1.63E+07	2.00E-03	-1.00E+00	3.20E-03
		-0.37	0.00	0.03	4.37E+09	-9.96E-01	-1.70E-03	8.60E-02
D	M1	20.85	3.09	-20.62	2.17E+08	-1.46E-01	-9.47E-01	-2.88E-01
		3.09	5.56	-19.14	1.45E+10	9.41E-01	-2.23E-01	2.56E-01
		-20.62	-19.14	73.61	8.53E+10	-3.07E-01	-2.33E-01	9.23E-01
	M2	17.78	3.00	-18.36	1.96E+08	-1.41E-01	-9.47E-01	-2.89E-01
		3.00	5.57	-19.03	1.27E+10	9.50E-01	-2.11E-01	2.29E-01
		-18.36	-19.03	71.38	8.18E+10	-2.78E-01	-2.43E-01	9.29E-01

Table 2. DoE optimality criteria for models M1 and improvement of using M1 instead of M2.

Mode	Model	Functions of FIM			
		A-optimal	D-optimal	E-optimal	ME-optimal
		Trace	Determinant	Minimal eigenvalue	Condition number
F	M1 Improvement	4.73E+09 8%↑	4.04E+22 17%↑	4.93E+05 2%↑	9551 6%↑
	M2	4.38E+09	3.45E+22	4.85E+05	9002
D	M1 Improvement	1.00E+11 6%↑	2.69E+29 32%↑	2.17E+08 11%↑	393 6%↓
	M2	9.47E+10	2.03E+29	1.96E+08	418

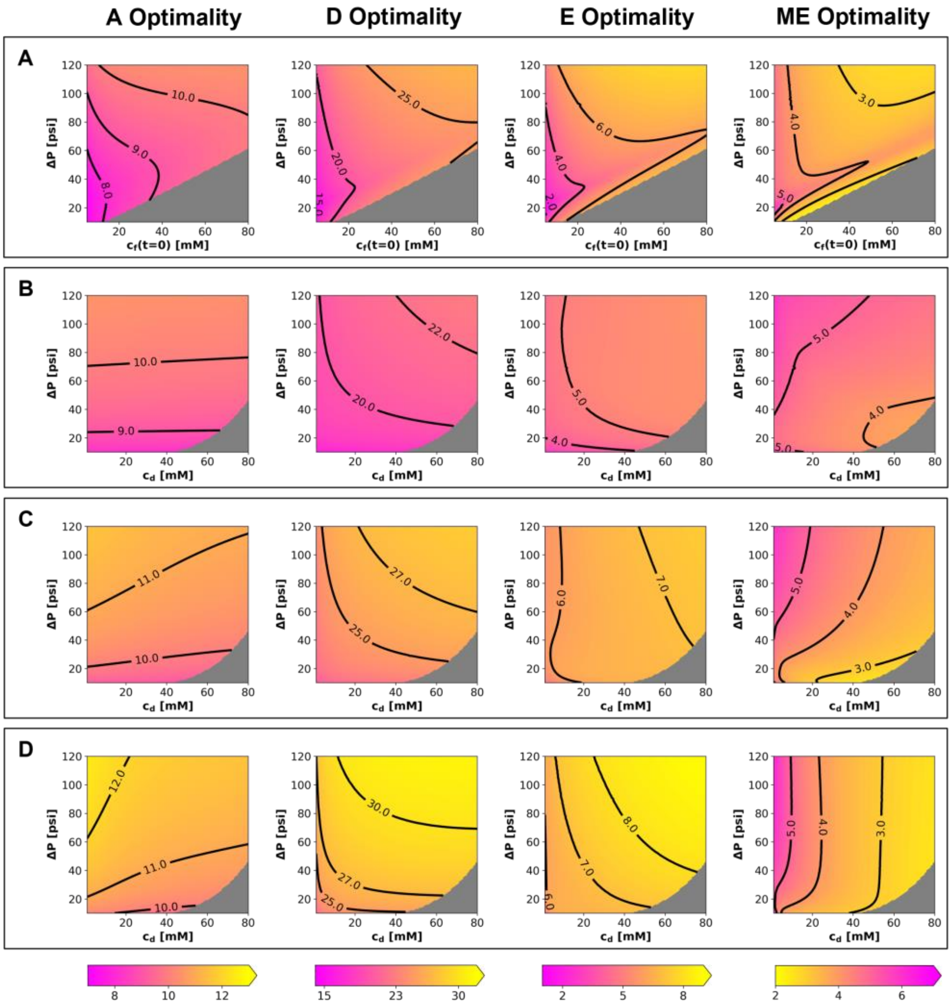

Fig. 1. A-, D-, E-, ME-optimal criteria evaluated under varying experimental conditions. Panel A shows filtration experiment (with 8 vials collected) predictions for varying initial feed concentration and applied pressure. Panels B, C, D examine the diafiltration experiment with 1, 5 and 10 vials collected, respectively, for the diafiltrate concentration and applied pressure.

4.3. MBDoE optimizes DATA system

We now use A-, D-, E-, and ME-optimality criteria to inform the applied pressure, initial retentate or diafiltrate concentrations (experimental design decisions) necessary to identify all parameters in filtration and diafiltration. Fig. 2A. examines filtration experiment at varying initial feed concentration $c_f(t=0)$ and applied pressure ΔP with 8 vials collected. Fig. 2B, 2C, and 2D examine diafiltration experiments at varying diafiltrate concentration c_d with 1, 5, and 10 vial collected, respectively. The gray regions correspond to physically impossible operating conditions where the water flux is equal to or less than zero. The contour lines show the $\log_{10}$-transformed values of every criterion. Comparing Fig. 2C to 2A, the lighter color and larger contour values for A-, D-, E-optimality metrics indicates that the diafiltration experiments with 5 vial collections contains more information than the filtration experiment with 8 vial collections. Moreover, higher applied pressures maximize A-, D-, and E-optimal metrics. However, based on ME-optimality, low applied pressure is desired in diafiltration experiment with 5 or fewer vial collections to remove the correlation among parameters. Increasing to 10 vial collections in diafiltration, shown in Fig. 2D, resolves the trade-off between parameter precision (A and D) and removing correlations (ME). Thus, with 10 vial collections, diafiltration experiments with a feed concentration of 5 mM KCl should be performed with a diafiltrate concentration greater than 50 mM KCl and an applied pressure at least 45 psi to identify all parameters with an order of magnitude of improvement in precision over filtration experiments.

5. Conclusions

In this paper, we apply MBDoE analyses to quantify the information gain in a recently proposed diafiltration apparatus for high-throughput analysis (DATA) for membrane characterization. In the future, MBDoE can be used to discriminate possible phenomena and mechanisms within complex multi-component systems and optimize diafiltration experiments with more degrees of freedom (e.g., time-varying applied pressure).

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