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Making Money in Energy Markets: Probabilistic Forecasting and Stochastic Programming Paradigms

Xian Gao[1] and Alexander W. Dowling[1]

Abstract— **Undoubtedly, evolving wholesale electricity markets continue to provide new revenue opportunities for diverse generation, energy storage, and flexible demand technologies. In this paper, we quantitatively explore how price uncertainty impacts optimal market participation strategies and resulting revenues. Specifically, we benchmark 2-stage stochastic programming formulations for self-schedule and bidding market participation modes in a receding horizon model predictive control framework. To generate probabilistic price forecasts, we propose an autoregressive Gaussian process regression model and compare three sampling strategies. As an illustrative example, we study a price-taker generation company with six unique generation units using historical price data from CAISO (California market). We show that self-schedule is sensitive to the error in the forecast mean, whereas bidding requires price forecasts that cover extreme events (e.g., tails of the distribution). We benchmark realized market revenue against optimal bidding with perfect information and find static bid curve, time-varying bid curve, and self-schedule modes recovery 95.29%, 94.85%, and 84.87% of perfect information revenue, respectively.**

I. INTRODUCTION

In the modern smart grid paradigm, hierarchical markets, including the day-ahead markets (DAM), the real-time markets (RTM), the ancillary service markets, ensure that the electricity demand and supply are synchronized by coordinating a diverse set of energy systems [1], [2]. Moreover, market transaction volumes have increased to help balance the increased integration of intermittent and renewable resources. Participation in the market provides electrical energy generation companies (GenCos), energy-intensive industrial systems, energy storage systems, and new technologies like hybrid systems great revenue opportunities by exploiting advanced control algorithms [2]. A standard technique to quantify economic opportunities for these technologies is to calculate the maximum possible revenue in retrospect [3]. Recently, several retrospective studies consistently find the greatest revenue opportunities from fast market timescales (e.g., real-time markets) and ancillary service products (e.g., frequency regulation, reserves) [1], [4], [5], [6]. These retrospective analyses all assume perfect information, i.e., ignore price, weather, and other sources of market uncertainty, and thus only provides an upper bound on revenue opportunities. In reality, resources participate in the wholesale energy markets under uncertainty via two modes: self-schedule and bidding.

When a resource self-schedules into the market, it determines when and how much electricity to generate/consume as a function of time. This decision can be formulated as a single or multi-stage stochastic program where resources seek to minimize their expected operational cost or maximize expected revenues. Future energy prices are treated as uncertain parameters, where price forecasts are used to generate scenarios. Autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) are the most popular price forecasting methods [7], [8], [9], [10]. Self-schedule examples in literature include thermal generators [11] and energy-intensive industrial separations [8]. Recently, Kumar *et al.* studied the payback period of a stationary battery system under load and price uncertainty. They proposed a Ledoit-Wolf covariance estimator to generate scenarios for a 2-stage stochastic program [12]. Baringo *et al.* demonstrated self-schedule strategies for a virtual power plant, which included a conventional power plant, energy storage, wind generations, and a demand unit, under wind production and price uncertainties [13].

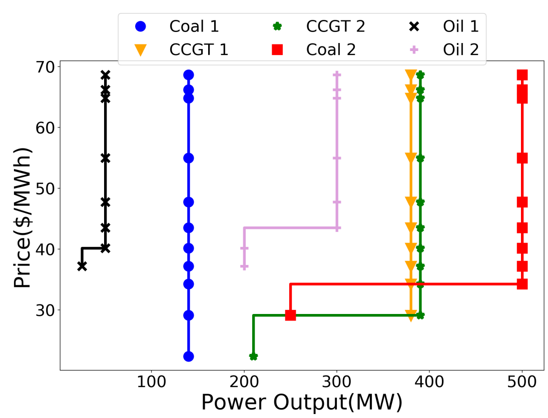

Fig. 1. Bid curves for the six thermal generators (Coal 1, Coal 2, CCGT1, CCGT2, Oil 1, Oil 2) in the illustrative case study. Each bid curve communicates the generator's ability (or willingness) to provide different amounts of electric power generation (MW) as a function of market clearing price ($/MWh). These specific time-varying bid curves were generated with the contour sampling technique using forecasts for CAISO prices at 4pm on Jan. 24th, 2015. See Sections II and III for details.

Alternatively, an energy resource submits bid curves to the market. Bid curves are piece-wise constant price and power pairs, as shown in Fig. 1. A bidding curve communicates to the market the resource's flexibility and marginal costs.

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[1]Xian Gao and Alexander W. Dowling are with the Department of Chemical & Biomolecular Engineering, University of Notre Dame, Notre Dame, IN 46556, USA {xgao1,adowling}@nd.edu

Calculation of time-varying bidding curves for the DAM and RTM is also formulated as stochastic programs, and again the expected operational cost (revenue) is minimized (maximized). A non-decreasing constraint, an analog to a non-anticipativity constraint, enforces the shape. For example, a 2-stage stochastic program is proposed to derive a bidding strategy for thermal generators [7] and energy-intensive aluminum smelter [14]. Stochastic programs have also been applied to derive bidding curves for renewable energy systems such as wind power and concentrating solar power plants where renewable energy input is uncertain [9], [10].

In the context of the above summarized prior work, this paper makes several contributions. Instead of ARIMA and other popular price forecasting methods (see Weron for a detailed literature review [15]), we propose an autoregressive Gaussian process regression method to generate *probabilistic* price forecasts. We then quantify revenue opportunities for self-schedule and bidding models in a model predictive control (MPC) receding horizon framework for an entire year using historical data from the California Independent System Operator (CAISO) market. Finally, we discuss how three different scenario generation strategies impact the profitability of self-schedule and bidding modes. Section II presents Gaussian process regression and stochastic programming mathematical models. Section III presents results from an illustrative case study. Finally, Section IV presents conclusions and directions of future work.

II. MATHEMATICAL FORMULATION

A. Gaussian Process Regression for Price Forecasts

We start by describing the Gaussian process regression framework using notation from Bishop [16]. We postulate a Gaussian distribution with a zero mean and a covariance $\boldsymbol{K}$ constructed by the kernel function $k(\cdot,\cdot)$ as a prior distribution for the underlying energy price function $\boldsymbol{y}$:

$$p(\boldsymbol{y}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{0},\boldsymbol{K}) \ , \tag{1}$$

where $\boldsymbol{K}$ is the Gram matrix with elements $\boldsymbol{K}_{mn} = k(\boldsymbol{x}_m,\boldsymbol{x}_n)$ and $\boldsymbol{x}$ is the input to the Gaussian process with D dimensions. We select a radial-based function (RBF) with automatic relevance determination (ARD) for the kernel [17]:

$$k(\boldsymbol{x}_m,\boldsymbol{x}_n) = \sigma_f^2 \exp[-\frac{1}{2}\sum_{i=1}^{D}\frac{(x_{mi}-x_{ni})^2}{l_i^2}] \ , \tag{2}$$

with length scales l_i and variance σ_f^2. Given additive "observation" noise ε, we have the relationship between the realized price π and the underlying price function y:

$$\boldsymbol{\pi} = \boldsymbol{y} + \boldsymbol{\varepsilon} \tag{3}$$

This leads to a Gaussian likelihood for the N observed historical prices $\boldsymbol{\pi}$:

$$p(\boldsymbol{\pi}|\boldsymbol{y}) = \mathcal{N}(\boldsymbol{\pi}|\boldsymbol{y},\beta^{-1}\boldsymbol{I}_N) \tag{4}$$

where β is the precision of the "observation" noise. Using the properties of linear Gaussian models, one can derive the marginal likelihood function [16]:

$$p(\boldsymbol{\pi}) = \int p(\boldsymbol{\pi}|\boldsymbol{y})p(\boldsymbol{y})d\boldsymbol{y} = \mathcal{N}(\boldsymbol{\pi}|\boldsymbol{0},\boldsymbol{C}) \tag{5}$$

where $\boldsymbol{C} = \boldsymbol{K} + \beta^{-1}\boldsymbol{I}_N$. The marginal likelihood computed in Eq. (5) assumes the observed prices $\boldsymbol{\pi}$ follow a joint Gaussian distribution. This assumption extends to an unobserved future price π^* at a new input $\boldsymbol{x}^*$:

$$p(\boldsymbol{\pi}^*) = \mathcal{N}(\boldsymbol{\pi}^*|\boldsymbol{0},\boldsymbol{C}^*) \ . \tag{6}$$

The extended covariance matrix $\boldsymbol{C}^*$ becomes,

$$\boldsymbol{C}^* = \begin{pmatrix} \boldsymbol{C} & \boldsymbol{k} \\ \boldsymbol{k}^T & c \end{pmatrix} \ , \tag{7}$$

where $\boldsymbol{k}$ is a vector of $k(\boldsymbol{x}_n,\boldsymbol{x}^*)$ for $n = 1,...,N$ and c is a scalar $k(\boldsymbol{x}^*,\boldsymbol{x}^*)$. We then marginalize the joint distribution, Eq. (6), to derive the predictive distribution:

$$p(\pi^*|\boldsymbol{\pi}) = \mathcal{N}(\pi^*|m(\boldsymbol{x}^*),\sigma^2(\boldsymbol{x}^*)) \tag{8}$$

The predictive mean and variance functions depend on the input $\boldsymbol{x}^*$ through $\boldsymbol{k}$,

$$\begin{aligned} m(\boldsymbol{x}^*) &= \boldsymbol{k}^T\boldsymbol{C}^{-1}\boldsymbol{\pi} \ , \\ \sigma^2(\boldsymbol{x}^*) &= c - \boldsymbol{k}^T\boldsymbol{C}^{-1}\boldsymbol{k} \ . \end{aligned} \tag{9}$$

In this paper, to predict the wholesale price at hour h, the input $\boldsymbol{x}_h^* = [\pi_{h-D},...,\pi_{h-1}]$ is a vector of the immediate $D = 72$ previous prices. The predictive output π_h^* corresponds to the realized price π_h. To train the GP model, 3 weeks of DAM energy price data ($N = 504$) were used as the training set $\boldsymbol{\pi}$. The marginal likelihood Eq. (5) is maximized to learn the hyperparameters length scales l_i, kernel variance σ_f^2, and likelihood noise precision β.

Because the DAM market schedules 24 hours at a time (midnight to midnight), price forecasts for over 24 hours need to be generated at once. Recall, however, the GP model only forecasts the price at the next timestep. We propose Algorithm 1 to generate a forecast with T timesteps.

Algorithm 1 Autoregressive GP Regression Price Forecast

for $s = 0$ to $N_s - 1$ **do**
  Initialize forecast scenario $\boldsymbol{\pi}_s^* = [\ ]$
  Initialize predictive input $\boldsymbol{x}^* = [\pi_{-D},...,\pi_{-1}]$
  for $h = 0$ to $T-1$ **do**
    Use $\boldsymbol{x}^*$ to compute the predictive distribution $p(\pi^*|\boldsymbol{\pi})$ via Eq. (8)
    Sample π_h^* from $p(\pi^*|\boldsymbol{\pi})$ with strategies from Section III-A
    Append $\pi_h^* \rightarrow \boldsymbol{\pi}_s^*$, delete $\boldsymbol{x}^*[0]$, append $\pi_h^* \rightarrow \boldsymbol{x}^*$
  end for
end for

B. Thermal Generator Portofolio

For demonstration, we consider a generation company (GenCo) which has 6 generation units, i.e. Coal 1, CCGT 1. CCGT 2, Coal 2, Oil 1, and Oil 2. The characteristics of the generators can be found in Table I [7]. A linear cost function is considered for these units and a static bid curve can be derived from this cost function.

C. Stochastic Programming for Self-schedule

With the model described above, we can formulate the day-ahead self-schedule problem as the following stochastic mixed-integer linear program (MILP).

$$\max \quad \frac{1}{||\mathscr{S}||} \sum_{j\in\mathscr{G},h\in\mathscr{T},s\in\mathscr{S}} (\pi^*_{hs} q_{jhs} - c_j q_{jhs} - c^F_j y_{jhs}) \qquad (10)$$

s.t.

$$q_{jhs} = \frac{1}{2}(p_{j(h-1)s} + p_{jhs}) \quad \forall j,h,s \qquad (11)$$

$$P^{min}_j y_{jhs} \le p_{jhs} \le P^{max}_j y_{jhs} \quad \forall j,h,s \qquad (12)$$

$$p_{jhs} \le p_{j(h-1)s} + R^{up}_j y_{j(h-1)s} + R^{SU}_j (y_{jhs} - y_{j(h-1)s}) + P^{max}_j (1 - y_{jhs}) \quad \forall j,h,s \qquad (13)$$

$$p_{j(h-1)s} - p_{jhs} \le R^{dw}_j y_{jhs} + R^{SD}_j (y_{j(h-1)s} - y_{jhs}) + P^{max}_j (1 - y_{j(h-1)s}) \ \forall j,h,s \qquad (14)$$

$$p_{jhs} \le P^{max}_j y_{j(h+1)s} + R^{SD}_j (y_{jhs} - y_{j(h+1)s}) \quad \forall j,h,s \qquad (15)$$

$$\sum_{h=0}^{G_j-1} (1 - y_{jhs}) = 0 \quad \forall j,s \qquad (16)$$

$$\sum_{h=h_0}^{h_0+UT_j-1} y_{jhs} \ge UT_j (y_{jh_0s} - y_{j(h_0-1)s}) \quad \forall j,s \quad \forall h_0 \in [G_j,...,T-UT_j] \qquad (17)$$

$$\sum_{h=h_0}^{T-1} [y_{jhs} - (y_{jh_0s} - y_{j(h_0-1)s})] \ge 0 \quad \forall j,s \quad \forall h_0 \in [T-UT_j+1,...,T-1] \qquad (18)$$

$$\sum_{h=0}^{L_j-1} y_{jhs} = 0 \quad \forall j,s \qquad (19)$$

$$\sum_{h=h_0}^{h_0+DT_j-1} (1 - y_{jhs}) \ge DT_j (y_{j(h_0-1)s} - y_{jh_0s}) \quad \forall j,s \quad \forall h_0 \in [L_j,...,T-DT_j] \qquad (20)$$

$$\sum_{h=h_0}^{T-1} [1 - y_{jhs} - (y_{j(h_0-1)s} - y_{jh_0s})] \ge 0 \quad \forall j,s \quad \forall h_0 \in [T-DT_j+1,...,T-1] \qquad (21)$$

This model uses three 3 indices and sets: generation units $j \in \mathscr{G} :=$ {Coal 1, CCGT 1, CCGT 2, Coal 2, Oil 1, Oil 2}, hours $h \in \mathscr{T} := [0,1,...,T-1]$ where T is the length of the planning horizon, and scenarios $s \in \mathscr{S}$. Unless otherwise noted, $\forall j$ implies $\forall j \in \mathscr{G}$, $\forall s$ implies $\forall s \in \mathscr{S}$, and $\forall h$ implies $\forall h \in \mathscr{T}$ in Eqs. (10) - (21). Decision variables are the amount of energy generated q_{jhs}, the generation power output p_{jhs}, and the on/off binary indicator y_{jhs} for each unit. Parameters are given in Table I.

The GenCO seeks to maximize its expected net revenue from all six units participating in the DAM. The first term in Eq. (10) calculates the expected revenue using price forecasts π^*_{hs}. Production and fixed costs are captured in the second and third terms, respectively. The problem is subject to the physics of the generators. Eq. (11) states that the energy delivered during $(h-1,h]$ is the average power at time $h-1$ and h. Eq. (12) bounds on the power outputs. Eq. (13)-(15) enforces ramp-up, ramp-down, start-up, and shutdown ramp rate limits [18]. Moreover, the generators are constrained by the minimum uptime over the whole planning horizon [18]: if the initial margin to the minimum uptime $G_j = (UT_j - UT^{init}_j) y^{init}_j$ is positive, Eq. (16) requires the generators to be online in the initial G_j hours, otherwise this constraint is skipped; Eq. (17) constrains the generators to satisfy the minimum uptime limit in the subsequent sets of consecutive UT_j hours; and Eq. (18) demands the generators online until the end of the planning horizon if they are started up in the last $UT_j - 1$ hours in the horizon. Similarly, minimum downtime constraints Eq. (19) - (21) apply to the entire planning horizon [18].

TABLE I

CHARACTERISTICS OF THE THERMAL UNITS [7]

Units	Coal1	CCGT1	CCGT2	Coal2	Oil1	Oil2
Fixed Cost c^F_j (\$/h)	126.0	1097.2	992.8	575.0	91.5	1800.0
Production Cost c_j (\$/MWh)	19.81	25.17	25.51	29.37	37.91	33.91
Maximum Power P^{max}_j (MW)	140	380	390	500	50	300
Minimum Power P^{min}_j (MW)	75	160	180	250	25	200
Ramp-up Limit R^{up}_j (MW/h)	65	220	210	250	25	100
Ramp-down Limit R^{dw}_j (MW/h)	65	220	210	250	25	100
Start-up Ramp Limit R^{SU}_j (MW/h)	75	220	210	250	25	200
Shut-down Ramp Limit R^{SD}_j (MW/h)	75	220	210	250	25	200
Minimum Uptime UT_j (h)	2	1	2	4	1	3
Minimum Downtime DT_j (h)	2	1	2	4	1	3

The non-anticipativity constraint, Eq. (22), enforces that the stochastic program does not "see into the future". In the first stage, hour $h \in [0, T'-1]$, the power output in each scenario is unanimous and independent of scenarios. But in the second stage ($h \in [T', T-1]$), the power outputs are dependent of the scenarios. This models the GenCo's ability to react in the second stage to realized uncertainty.

$$p_{jhs} = p_{jhs'},\ \forall j \in \mathcal{G},\ s \in \mathcal{S},\ s' \in \mathcal{S} \setminus s,\ h \in [0, T'-1] \quad (22)$$

Solving the stochastic program, Eqs. (10) - (22) gives the optimal energy production $q_{jhk} \forall h \in \mathcal{T}$ and power output $p_{jhk} \forall h \in \mathcal{T}$ schedule for the trading hours in the next day. We record this schedule, and then compute the actual revnues using the realized price $[\pi_0, \pi_{T'-1}]$ (historical data). The time horizon is then advanced and as expected in the rolling horizon framework, we update the initial uptime UT_j^{init} and the initial downtime DT_j^{init} and then resolve Eqs. (10) - (22).

D. Stochastic Programming for Bidding

The day-ahead bidding problem is similar to the self-schedule problem. Specifically, the bidding problem uses the same objective, Eq. (10), and physical constraints, Eqs. (11) - (21). The non-anticipativity constraint is replaced with a constraint to ensure the bidding curve (e.g., Fig. 1) is monotonically increasing, i.e. production increases with price:

$$(p_{jhs} - p_{jhs'})(\pi^*_{hs} - \pi^*_{hs'}) \geq 0,$$
$$\forall j \in \mathcal{G},\ s \in \mathcal{S},\ s' \in \mathcal{S} \setminus s,\ h \in [0, T'-1] \quad (23)$$

After solving the stochastic program, the time-varying bidding curves for each hour of a day can be derived. Each point on the bidding curve corresponds to a price forecast scenario in the stochastic program. Simulating bidding in a rolling horizon is more nuanced than self-schedule. Using the generated bid curves, we assume the GenCo is a price-taker and solve a simple market clearing problem. Based on the schedule from the cleared market, we solve another optimization problem with the same physical constraints to track the set points. Then, like self-schedule, we record the settlement with the realized prices. Similarly, the initial uptime UT_j^{init} and the initial downtime DT_j^{init} are updated in the model and then we advance to the next day.

III. RESULTS

A. Price Forecasts & Sampling Methods

After training the GP price forecast model, we have the predictive Gaussian distribution $p(\pi^*|\boldsymbol{\pi})$ from Eq. (8) and the predictive mean $m(\boldsymbol{x}^*)$ and variance functions $\sigma^2(\boldsymbol{x}^*)$ from Eq. (9). We then use Algorithm 1 to make probabilistic DAM price forecasts with three sampling strategies:

1) **Monte Carlo Sampling**
We generate Monte Carlo (MC) samples directly from the predictive distribution $p(\pi^*|\boldsymbol{\pi})$. As shown in Fig. 2 (top), this sampling method provides scenarios that closely follow the forecast mean. MC sampling captures the realized prices well with a root mean squared error (RMSE) of \$4.59/MWh and a typically narrow forecast range from \$30.25/MWh to \$37.34/MWh.
2) **Uniform Sampling**
Alternatively, we may want to emphasize on the tail of the Gaussian distribution, i.e., sample farther from the mean $m(\boldsymbol{x}^*)$. To do this, we generate uniform samples from $\mathcal{U}(m(\boldsymbol{x}^*) - 3\sigma(\boldsymbol{x}^*), m(\boldsymbol{x}^*) + 3\sigma(\boldsymbol{x}^*))$. Fig. 2 (middle) shows greater variability than the MC samples. As expected, uniform sampling gives estimates with a large RMSE (\$5.30/MWh) and an intermediate forecast range from \$26.48/MWh to \$39.83/MWh.
3) **Contour Sampling**
To further emphasize the probability density tail, we generate deterministic samples by computing $m(\boldsymbol{x}^*) + a_i\sigma(\boldsymbol{x}^*)\ \forall i \in [1, ..., 13]$ where a_i are 13 evenly spaced fixed values between $[-3, 3]$. Fig. 2 (bottom) shows that the Contour Sampling emphasizes extreme forecasts the most, and has the highest RMSE of \$13.30/MWh and also largest range from \$9.75/MWh to \$58.69/MWh.

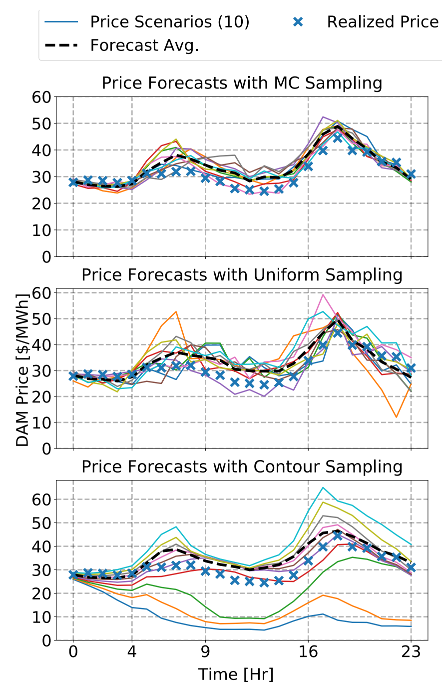

Fig. 2. GP price forecasts comparison among different sampling strategies on Jan. 24th, 2015. **Top**: Price forecast scenarios generated by Monte Carlo Sampling and the scenarios concentrate around the average; **Middle**: Price forecast scenarios generated by Uniform Sampling and the scenarios fluctuate and deviate from the average; **Bottom**: Price forecast scenarios generated by Contour Sampling, which capture overall increasing and decreasing trends but greatly deviate from the forecast mean.

B. Market Participation under Uncertainty

We now compare combinations of the three sampling strategies (MC, uniform, contour) and three stochastic market participation modes (self-schedule, time-varying bid curve, static bid curve). Fig. 2 shows price forecasts for Jan. 24, 2015 and Fig. 3 shows the power output profiles of each market participation mode with the sampling strategy that captures the greatest revenue (with realized prices). We benchmark each approach against optimization with perfect information (no uncertainty).

From **0 am - 3 am**, the realized prices are stable, which has a mean of \$28.17/MWh with a small standard deviation of \$0.41/MWh. And the perfection information model dispatches Coal 1, CCGT 1 and CCGT 2 at full capacity (Fig. 3, lower right). But because the forecast mean

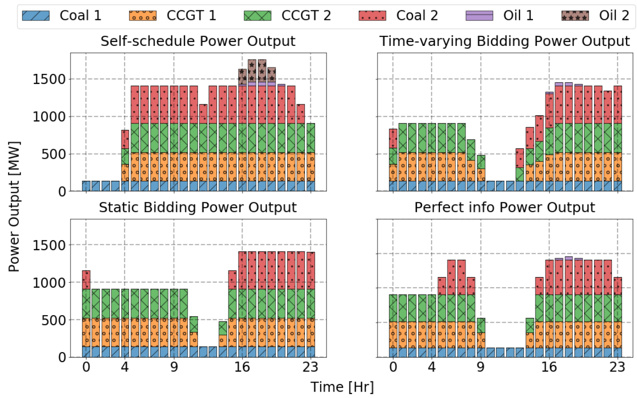

Fig. 3. Power output comparison among different market participation strategies on Jan. 24th, 2015. Upper Left: Power outputs of self-schedule using Monte Caro sampling method and the power outputs depend on the trend of the average forecasts; Upper Right: Power outputs of bidding curves with Contour sampling which overcomes the dependencies on the average of the forecasts; Lower Left: Power outputs of static bidding curves are insensitive to price forecasts and instead capture the production costs of the generators; Lower Right: Power output of perfect information model which serves as the benchmark.

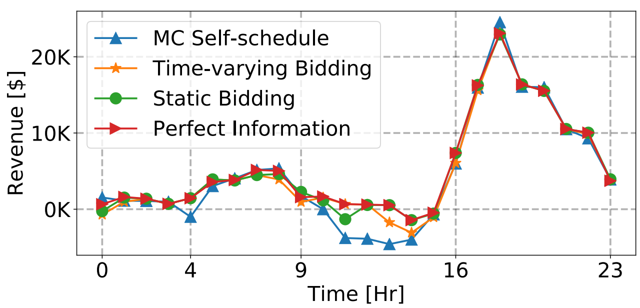

Fig. 4. Hourly revenue comparison among different market participation strategies on Jan. 24th, 2015.

is below the realized price, for self-schedule, this causes only the generator Coal 1, whose production cost is the lowest, to be dispatched (Fig. 3, upper left). Both bidding curve approaches are more robust to this forecasting error and have smaller power output deviations from perfect information, as market-clearing is performed using the realized price. In Fig. 3 upper right, slight deviation from the perfect information model at 0 am regarding Coal 2 is because of the temporal ramp-down limit constraints, Eqs. (14) and (15), from the previous day (not shown in the figure).

As for **4 am - 8 am**, there is an increase in the realized DAM energy price from $28.51/MWh to $32.00/MWh. The perfect information model exploits this opportunity by dispatching Coal 2 and then ramp down at 8 am in anticipation of future prices. The forecast averages of MC sampling capture that the price is going up at 4 am. As such, the self-schedule model begins increasing power for Coal 2 at 4 am but not as high as the perfect information model due to the ramp-up limit (Eq. (13)). Later, the self-schedule increases production for Coal 2 even more than the perfect information model because the forecasted mean price is above the realized price. Neither bidding curve models change dispatch from 4 - 8 am because of simplifications in our market simulator (e.g., no start-up costs, no uplift payments) and the temporal ramp-up limit constraint (Eq. (13)).

From **9 am - 3 pm**, there is a decrease in the market prices from 9 am to 1 pm from $29.42/MWh to $24.52/MWh which resulted in instantaneous revenue losses. Accordingly, the perfect information model turns off all units except Coal 1. For 2 - 3 pm, CCGT1 and CCGT 2 are dispatched in anticipation of the later prices increase, even with temporary negative revenues. With the self-schedule model, though the forecast mean predicts that there is a decrease from 9 am to 12 pm, the drop is not big enough to shut down CCGT 1, CCGT 2 and Coal 2 which resulted in the largest revenue loss in the day (shown in Fig. 4). The time-varying bidding curve model can lower the power output from 9 am to 12 pm. But due to the market-clearing results and ramp-up limit, the ramp-up of CCGT 2 and Coal 2 increases the power again at 1 pm which is earlier than the perfect information model. This premature ramp-up also leads to a decrease in revenue. In contrast, because the realized prices at 9 am and 10 am are higher than the production cost of the CCGT 1 and CCGT 2, in the static bidding curve model the units bid and were scheduled for max power output. This delays shutting down the 2 units and then a loss of revenue at 11 am.

Lastly, during **4 pm - 11 pm**, the realized market price starts to climb at 4 pm and reaches its peak at 6 pm at a value of 44.46$/MWh. Consequently, the perfect information model increases the total power by dispatching Coal 1, Coal 2, CCGT 1 and CCGT 2 at full capacity. Even Oil 1, which has the highest production cost of 37.91$/MWh, is dispatched. But because the MC sampling average overestimates the prices, all the units, including Oil 2 who has the highest fixed cost of 1800$/hr and the second-highest production cost of 33.91$/MWh, are dispatched at their full capacity at 5 pm and 6 pm. In contrast, both bidding models are conservative about dispatching Oil 1 and Oil 2. On the one hand, for the time-varying bidding model, although Oil 2 bid at least minimum power between 4 pm and 8 pm, the market dispatches only Oil 1 but not Oil 2. We suspect this is because of our simplified market-clearing algorithm (e.g., no uplift payments). On the other hand, the static bidding curve model did not even utilize Oil 1. Though Oil 1 and Oil 2 put in their bids from 4 pm to 8 pm, due to the full utilization of the cheaper unit Coal 2 at 4 pm and temporal ramp limit constraints, Oil 1 and Oil 2 are not dispatched. After 6 pm, the revenues captured by all the models are similar.

Based on the detailed analysis of Fig. 3, we observe several **general trends**. First, the self-schedule model is most sensitive to price forecasting errors. Specifically, large deviations between the average price forecast and true (realized) price can cause significant deviations from the perfect information schedule. For example, from 9 am - 3 pm, the MC forecasts overestimate the prices which results in the largest instantaneous revenue loss on this day as shown in Fig. 4. In contrast, both static and time-varying bidding are more robust to price uncertainties. As a benchmark, the perfect information model gives $129,458 in revenue, which follows static bidding $125,816 (97.19% of perfect information), time-varying bidding with contour sampling $118,661 (91.66%), and self-schedule with Monte Carlo sampling $108,384 (83.72%).

TABLE II

ANNUAL REVENUE OPPORTUNITIES COMPARISON

Uncertainty	Bidding Curve	Self-schedule
Perfect Information	$70,568,283 (100%)	
Static Bidding Curve	$67,241,092 (95.29%)	-
MC Sampling	$54,205,759 (76.81%)	$59,889,350 (84.87%)
Uniform Sampling	$57,055,508 (80.85%)	$58,962,994 (83.55%)
Contour Sampling	$66,933,688 (94.85%)	$58,762,205 (83.27%)

Next, we compare eight forecasting and bidding strategies by computing revenues over all of the calendar year 2015. Table II summarized the results. The perfect information model is used as a benchmark, recording $70.5M in annual revenue. Because of weak temporal constraints and a linear cost function in the generator models, the static bidding curve model captures 95.29% of the available revenue. For time-varying bidding, we find contour sampling captures the most revenue (94.85%) and MC sampling captures the least (76.81%). For self-schedule, we find the opposite trend: MC sampling captures the largest revenue (84.87%) and contour sampling captures the least (83.27%). Computational results were obtained using Gurobi 8.1.1 and Pyomo on a 24-core Linux-based server (2.50 GHz clock speed, 251 GB of RAM). Simulating 343 days took 1.8 hours on average.

IV. CONCLUSIONS

Participation in the modern wholesale energy market provides generation companies (GenCos) and energy-intensive industrial systems with great revenue opportunities. In this paper, we propose an autoregressive Gaussian process regression forecasting model. As future work, we plan to benchmark our GP method against other forecasting techniques. In this paper, we quantitatively compare the economic opportunities brought by self-schedule and bidding under energy price uncertainty with 3 different sampling strategies: Monte Carlo sampling, uniform sampling, and contour sampling. We use a 6-unit GenCo who solely participates in the day-ahead energy market as an illustrative example. We find that self-schedule formulation is the most sensitive to forecasting errors. Monte Carlo sampling has the lowest root mean square forecasting error and is the best paired with self-schedule. On the other hand, the bidding curve mode is inherently robust to price uncertainty. Uniform and contour sampling are preferred as they bias extreme (tail probability) price scenarios. The time-varying bidding curves are more flexible than the static bidding curves, but they require sufficient scenarios that cover a wider price range. Although these conclusions are straightforward with thermal generators, energy storage systems offer new challenges from stronger temporal constraints. As future work, we plan to compare self-schedule and bidding curves for hybrid energy systems, which combines thermal generators and energy storage systems. We also plan to perform simulations with the high-fidelity market simulator Prescient.

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